Blackjack card analysis report

For PartyGaming system

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iTech Labs Australia ACN 108 249 761
Suite 24, 40 Montclair Ave, Glen Waverley, VIC 3150, Australia. Tel. +61 3 9561 9955 Fax: +61 3 9545 1596
www.itechlabs.com.au e-mail: info@itechlabs.com.au

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Executive Summary

iTech Labs have calculated theoretical probabilities for a number of outcomes in the Blackjack game and compared these against the probabilities for cards played by real money players. The calculations were done for 'Probabilities for specific events from the first two cards for Player and Dealer', 'Probabilities for the Dealer's final hand', 'First two cards for Player and Dealer' and 'Third and fourth cards for Player and Dealer'.

The calculations were done for a total of one million real money Blackjack hands. Our calculations for 'Probabilities for specific events from the first two cards for Player and Dealer' and 'Comparison of Player's and Dealer's first two cards' indicated statistical randomness. The observed values were within 95% confidence limits of the theoretical probabilities, except for minor exceptions that were very well within the acceptable the level of 5%.

iTech Labs have also done calculations for 'Probabilities for the Dealer's final hand', 'Comparison of Player's and Dealer's first two cards against expected values' and 'Comparison of Player's and Dealer's third and fourth cards against expected values'. The chi-squared statistic and p-value for these calculations showed statistical randomness.

In all the chi-squared tests performed, none of them was found to be statistically significant. There is thus no evidence of non-randomness in the sequence of cards drawn.

iTech Labs had previously tested and certified the Random Number Generator (RNG) used in the PartyGaming system. We continue to monitor the executables of the RNG. The findings from the analysis of cards from the live system confirms that the RNG used for the Blackjack game in the PartyGaming system is operating correctly and the random numbers are being used in a fair manner resulting in statistically random dealing of the cards.

1. Introduction

This report is aimed at providing theoretical probabilities and corresponding actual probabilities for the Blackjack game in the PartyGaming system. iTech Labs have calculated theoretical probabilities for a number of outcomes in the Blackjack game and compared these against the probabilities of cards played by real money players. The calculations were done for **Probabilities for specific events from first two cards for Player and Dealer, Probabilities for the Dealer's final hand, First two cards for Player and Dealer** and **Third and fourth cards for Player and Dealer.** These calculations are given in sections 2,3,4 and 5 respectively.

The actual probability calculations were done for a total of one million real money Blackjack hands from the PartyGaming system. The results of these calculations were compared against a much smaller number of hands captured by iTech Labs separately. In both cases, the results were within 95% confidence limits. All actual calculations for cards shown in this report are for the 1 million hands.

For calculations in sections 2 and 4, we have used only the first two cards dealt to the player and the dealer. Since the dealer strategy is fixed, we have used all cards drawn by the dealer for the calculations shown in section 3 **Probabilities for the Dealer's final hand.** We have used the third and fourth cards for calculations in section 5 **Third and Fourth cards for Player and Dealer.** Calculations in this section are applicable only if the player or dealer has chosen to draw a third card or a third card followed by a fourth card.

2. Probabilities for specific events from first two cards for Player and Dealer

The following table compares the observed proportion of times various events occurred (**Sample** column) against theoretical probabilities (**Probability** column). **Lower** and **Upper** are the 95% confidence limits.

Description	Probability	Lower	Sample	Upper
Dealer Blackjack	0.047451	0.047034	0.047330	0.047868
Player Blackjack	0.047451	0.047034	0.047191	0.047868
Dealer total 20 (from first 2 cards)	0.106024	0.105421	0.105627	0.106628
Player total 20 (from first 2 cards)	0.106024	0.105421	0.105595	0.106628
Dealer hole card is 10 value card (10-J-Q-K)	0.307692	0.306769	0.306931	0.308615

Calculations using live cards from PartyGaming system

Notes:

a) *In the above table, all numbers in the Sample column (observer probabilities) lie within 95% confidence limits.*

b) *Theoretical probability of 0.047451 for player blackjack means, players are likely to get blackjack in 4.74% of the total hands.*

c) *Example calculation for the value* 0.047451

With 8 packs there are 416 cards. There are 32 Aces and 128 cards worth 10, thus the probability of getting an Ace first card and 10,J,Q,K second is

$$\frac{32}{416}\cdot\frac{128}{415}.$$

Since it is also possible to get the 10,J,Q,K first we double this to get 0.047451.

d) *When the sample size is not small, the formula*

$$p-1.96\sqrt{\frac{p(1-p)}{n}}, p+1.96\sqrt{\frac{p(1-p)}{n}}$$

gives approximate 95% confidence limits. p comes from the probability column and n=100,0000 samples.

Example:

p	Sd = Sqrt(p(1-p)/1000000)	p-1.96xsd	p+1.96*sd
0.047451	0.000213	0.047034	0.047868

Comments:

All observed values lie within the 95% confidence limits.

3. Probabilities for the Dealer's final hand.

The following table compares theoretical probabilities against observed proportion of times for dealer's final hand for 17, 18, 19, 20, 21 and bust. **Probability** column shows the theoretical values. **Sample** column shows actual values. These calculations are applicable when dealer has 3 or more cards.

Dealer's Up card	17		18		19	
	Probability	Sample	Probability	Sample	Probability	Sample
Ace	0.1246	0.1222	0.1430	0.1425	0.1432	0.1442
2	0.1301	0.1307	0.1361	0.1366	0.1315	0.1319
3	0.1260	0.1257	0.1319	0.1340	0.1268	0.1273
4	0.1224	0.1217	0.1259	0.1250	0.1226	0.1244
5	0.1182	0.1173	0.1230	0.1236	0.1183	0.1184
6	0.1150	0.1156	0.1146	0.1148	0.1150	0.1134
7	0.0986	0.0984	0.0989	0.0968	0.1277	0.1276
8	0.0963	0.0984	0.0956	0.0962	0.0960	0.0938
9	0.0932	0.0966	0.0936	0.0960	0.0933	0.0921
10	0.0900	0.0909	0.0895	0.0890	0.0900	0.0893

Dealer's Up card	20		21		Bust	
	Probability	Sample	Probability	Sample	Probability	Sample
Ace	0.1437	0.1403	0.1433	0.1404	0.3022	0.3104
2	0.1257	0.1250	0.1199	0.1193	0.3567	0.3566
3	0.1221	0.1223	0.1163	0.1172	0.3769	0.3734
4	0.1178	0.1177	0.1131	0.1136	0.3981	0.3977
5	0.1128	0.1142	0.1087	0.1086	0.4191	0.4180
6	0.1102	0.1109	0.1058	0.1055	0.4393	0.4399
7	0.1280	0.1291	0.1202	0.1199	0.4265	0.4282
8	0.1289	0.1302	0.1292	0.1303	0.4540	0.4510
9	0.0933	0.0944	0.1315	0.1298	0.4951	0.4911
10	0.0896	0.0923	0.0901	0.0903	0.5509	0.5481
The chi-squared statistic was 47.5 on 50 degrees of freedom (df). Conclusion: do not reject randomness (i.e. accept cards were random) with p-value = 0.57.						

Notes:

a) The frequency of outcomes when the dealer has only 2 cards are shown in section 4.3.

b) In the above calculations, the dealer draws a card on soft 17.

c) The chi-squared statistic is a measure of how close the observed numbers are to the expected numbers. The calculation of the degrees of freedom is somewhat complicated but is always a bit less than the number of comparisons made. In the above two tables the number of comparisons was 60 but the degrees of freedom 50. The chi-squared statistic should be fairly close to the

degrees of freedom. If there are significant differences between the theoretical probabilities and the actual probabilities of hands produced, one would expect the chi-squared statistic to be very large when dealing with 1 million hands.

d) The p-value is the probability one would have got if the random number generator was working properly. In general no notice is taken of the p-value unless it is below 5% (0.05). 5% means that there is still a 1/20 chance that we would have got this data with the random number generator working properly. For 1 million hands even small deviations from randomness will show up as small p-values (say <0.001).

Comments:

The chi-squared statistic was 47.5 on 50 degrees of freedom (df). The p-value is 0.57 This indicates that the RNG is working correctly.

4. First two cards for Player and Dealer

4.1 Comparison of Player's and Dealer's first two cards

Using production data, we have calculated the distribution of the first two cards dealt to the player and the dealer. The following table compares the observed proportion of times the player and the dealer have received the cards shown in the **Cards** column. **Lower** and **Upper** are the 95% confidence limits.

Cards	Lower	Player cards	Upper	Lower	Dealer cards	Upper
Ace	76390	76725	77456	76390	76839	77456
2	76390	77089	77456	76390	76729	77456
3	76390	76620	77456	76390	76821	77456
4	76390	76717	77456	76390	77374	77456
5	76390	77262	77456	76390	76427	77456
6	76390	77091	77456	76390	76798	77456
7	76390	76797	77456	76390	77196	77456
8	76390	76625	77456	76390	76898	77456
9	76390	*77519*	77456	76390	76985	77456
10,J,Q,K	306769	307555	308615	306769	307933	308615

Calculations using cards from PartyGaming system

Notes:

a) The number shown in italics in the Player cards column is outside the Lower and Upper limits. It is acceptable for up to 5% of the observed numbers to lie outside the limits. One number (77519) is outside the lower and upper limits in the above table (equivalent to 5%).

b) Lower and Upper confidence limits are calculated as below:

$$np - 1.96\sqrt{np(1-p)}, np + 1.96\sqrt{np(1-p)}$$

where np is the expected number of cards.

np = probability (p) multiplied by 1000000 (n). For example, the probability the first card is an Ace is p=1/13; Total number of first cards n=1000000; expected number of first cards that are Aces np = (1/13) x 1000000

Comments: All observed probabilities except one were within 95% confidence limits. 1 in20 values can lie outside the range. This indicates that the RNG is working correctly.

4.2 Comparison of Player's first two cards against expected values

In the table below, un-shaded values are the expected values in 1 million hands, shaded values are the observed values (e.g. Ace was followed by 2 on 5951 occasions).

	Ace	2	3	4	5	6	7	8	9	10
Ace	5746	5931	5931	5931	5931	5931	5931	5931	5931	23725
	5787	5951	5963	5774	5930	6076	5901	5952	6046	23459
2	5931	5746	5931	5931	5931	5931	5931	5931	5931	23725
	5929	5781	5945	5879	5992	5827	5879	5963	5890	23644
3	5931	5931	5746	5931	5931	5931	5931	5931	5931	23725
	5867	5834	5567	5962	5929	5932	6081	5888	6019	23742
4	5931	5931	5931	5746	5931	5931	5931	5931	5931	23725
	5969	5987	6011	5798	6123	5878	5923	5947	5933	23805
5	5931	5931	5931	5931	5746	5931	5931	5931	5931	23725
	5978	5902	5822	5979	5665	5918	5822	5876	5876	23589
6	5931	5931	5931	5931	5931	5746	5931	5931	5931	23725
	6031	5915	6057	5855	5938	5757	5901	5951	5943	23450
7	5931	5931	5931	5931	5931	5931	5746	5931	5931	23725
	6005	6045	5828	5950	6072	5964	5751	6012	5984	23585
8	5931	5931	5931	5931	5931	5931	5931	5746	5931	23725
	6057	5914	5789	5970	5933	5949	5854	5882	5830	23720
9	5931	5931	5931	5931	5931	5931	5931	5931	5746	23725
	5894	5901	5848	5933	5908	5959	6089	5888	5728	23837
10	23725	23725	23725	23725	23725	23725	23725	23725	23725	94161
	23732	23424	23892	23914	24038	23648	24018	23680	23740	93847
Chi-s	7.12	8.79	19.07	8.43	15.57	6.51	15.40	5.90	6.61	9.91
Total chi-squared statistic = 103.3 on 99df **Do not reject randomness with p-value = 0.36.**										

Calculations using cards from PartyGaming system

Notes:

a) The cards J, Q and K are counted as 10.

Comments:

The chi-squared statistic was 103.3 on 99 degrees of freedom (df). The p-value is 0.36 This indicates that the RNG is working correctly.

4.3 Comparison of Dealer's first two cards against expected values

In the table below, un-shaded values are the expected values in 1 million hands, shaded values are the observed values (e.g. Ace was followed by 3 on 5957 occasions).

	Ace	2	3	4	5	6	7	8	9	10
Ace	5746	5931	5931	5931	5931	5931	5931	5931	5931	23725
	5703	5931	5957	5953	5942	5817	5882	5856	5963	23721
2	5931	5746	5931	5931	5931	5931	5931	5931	5931	23725
	6010	5833	5825	5980	6035	5969	5865	5949	5835	23788
3	5931	5931	5746	5931	5931	5931	5931	5931	5931	23725
	5940	5938	5782	5825	5885	5940	6011	5879	5932	23488
4	5931	5931	5931	5746	5931	5931	5931	5931	5931	23725
	5849	5898	5855	5759	5932	5996	5851	6009	5957	23611

5	5931	5931	5931	5931	5746	5931	5931	5931	5931	23725
	5996	6027	5947	5985	5742	6099	5948	5855	5892	23771
6	5931	5931	5931	5931	5931	5746	5931	5931	5931	23725
	5805	6016	6018	5943	5944	5743	5879	6019	5765	23959
7	5931	5931	5931	5931	5931	5931	5746	5931	5931	23725
	5809	5844	5885	5907	5865	5994	5873	5864	6001	23755
8	5931	5931	5931	5931	5931	5931	5931	5746	5931	23725
	5788	5767	5944	5837	5937	6082	5972	5737	5953	23608
9	5931	5931	5931	5931	5931	5931	5931	5931	5746	23725
	6079	5942	5979	6025	6098	5930	6011	5880	5930	23645
10	23725	23725	23725	23725	23725	23725	23725	23725	23725	94161
	23609	23505	23914	23859	23964	23713	23740	23707	23959	93585
Chi-s	16.13	12.15	6.81	6.76	10.08	12.44	7.98	5.99	15.85	9.89
Total chi-squared statistic = 103.3 on 99df **Do not reject randomness with p-value = 0.34.**										

Calculations using cards from PartyGaming system

Notes:

a) The cards J, Q and K are counted as 10.

Comments:

The chi-squared statistic was 103.3 on 99 degrees of freedom (df). The p-value is 0.34 This indicates that the RNG is working correctly.

4.4 Comparison of Player's opening score against Dealer's opening score

Using one million hands, Player's opening scores (2-20) is compared against Dealer's opening scores (2-20). To simplify the analysis, Ace is counted as 1. In the table below, un-shaded values are the expected values in 1 million hands, shaded values are the observed values.

Player	**Dealer**									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
2	--	*29*	64	97	133	166	202	235	271	304
	--	*45*	72	94	151	168	202	245	285	323
3	--	*64*	133	202	275	344	417	486	559	628
	--	*86*	130	188	298	370	410	501	611	638
4	--	97	202	300	408	510	619	721	830	932
	--	93	203	302	389	535	674	708	868	920
5	--	133	275	408	550	688	834	972	1118	1256
	--	150	271	446	567	661	811	993	1139	1192
6	--	166	344	510	688	855	1036	1208	1389	1561
	--	179	337	516	664	818	1061	1211	1440	1611
7	--	202	417	619	834	1036	1252	1458	1678	1885
	--	187	399	588	799	1045	1282	1432	1752	1815
8	--	235	486	721	972	1208	1458	1694	1949	2189
	--	221	491	745	931	1179	1504	1665	1937	2155
9	--	271	559	830	1118	1389	1678	1949	2238	2513
	--	257	583	838	1190	1367	1675	1988	2176	2458

10	--	304	628	932	1256	1561	1885	2189	2513	2818
	--	299	647	936	1244	1539	1864	2101	2531	2829
11	--	533	1114	1660	2242	2788	3370	3916	4498	5044
	--	547	1087	1642	2207	2769	3419	3832	4509	5092
12	--	515	1045	1556	2100	2611	3155	3665	4209	4720
	--	519	1049	1495	2033	2574	3238	3588	4296	4727
13	--	481	994	1458	1967	2444	2952	3430	3938	4416
	--	467	950	1475	1897	2439	2918	3497	3917	4411
14	--	446	921	1367	1824	2266	2737	3180	3650	4092
	--	426	937	1313	1886	2253	2746	3121	3723	4070
15	--	412	852	1265	1705	2100	2535	2943	3379	3788
	--	432	817	1305	1617	2119	2505	2917	3365	3819
16	--	377	779	1156	1558	1936	2320	2693	3090	3463
	--	384	775	1172	1503	1889	2329	2677	3223	3527
17	--	344	710	1054	1420	1765	2131	2457	2819	3159
	--	349	734	1052	1476	1794	2098	2482	2878	3176
18	--	308	637	945	1274	1583	1911	2220	2531	2835
	--	317	629	958	1312	1584	1966	2232	2545	2889
19	--	275	568	843	1136	1412	1705	1980	2273	2531
	--	287	580	872	1100	1409	1819	2069	2316	2504
20	--	546	1127	1674	2255	2802	3383	3929	4511	5057
	--	542	1189	1674	2258	2829	3453	3938	4513	4968
Chi-square		26.19	12.64	14.72	32.81	10.10	23.52	17.35	26.06	15.74

Player	**Dealer**									
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
2	533	515	481	446	412	377	344	308	275	546
	516	536	427	443	431	323	338	276	246	582
3	1114	1045	994	921	852	779	710	637	568	1127
	1083	1035	1038	910	826	789	723	641	578	1086
4	1660	1556	1458	1367	1265	1156	1054	945	843	1674
	1650	1541	1466	1424	1206	1185	1081	928	842	1715
5	2242	2100	1967	1824	1705	1558	1420	1274	1136	2255
	2202	2083	1971	1827	1749	1507	1399	1249	1120	2228
6	2788	2611	2444	2266	2100	1936	1765	1583	1412	2802
	2842	2557	2474	2289	2117	1885	1773	1638	1355	2831
7	3370	3155	2952	2737	2535	2320	2131	1911	1705	3383
	3349	3206	2968	2757	2461	2397	2126	1812	1718	3271
8	3916	3665	3430	3180	2943	2693	2457	2220	1980	3929
	3918	3699	3317	3169	3034	2745	2456	2152	2026	3923
9	4498	4209	3938	3650	3379	3090	2819	2531	2273	4511
	4494	4212	4058	3563	3347	3010	2870	2513	2214	4415
10	5044	4720	4416	4092	3788	3463	3159	2835	2531	5057
	5037	4653	4480	4194	3889	3538	3121	2868	2549	5016
11	8970	8441	7894	7313	6767	6185	5639	5057	4511	8952
	8857	8421	7961	7443	6755	6082	5633	5042	4631	8802

12	8441	7840	7382	6838	6327	5784	5272	4729	4218	8370
	8412	7700	7319	6846	6466	5768	5285	4776	4361	8154
13	7894	7382	6848	6392	5914	5406	4928	4420	3943	7824
	7826	7467	6853	6312	5965	5437	4932	4458	3945	7883
14	7313	6838	6392	5864	5475	5004	4563	4091	3649	7242
	7326	6908	6496	6038	5550	5068	4502	4058	3644	7369
15	6767	6327	5914	5475	5009	4626	4218	3783	3374	6696
	6708	6333	6011	5441	5000	4669	4307	3803	3452	6646
16	6185	5784	5406	5004	4626	4172	3852	3455	3081	6115
	6164	5813	5321	4989	4702	4155	3948	3567	3086	6197
17	5639	5272	4928	4563	4218	3852	3455	3146	2806	5568
	5698	5130	4893	4581	4266	3827	3375	3086	2774	5659
18	5057	4729	4420	4091	3783	3455	3146	2765	2513	4987
	4917	4751	4344	4111	3721	3475	3133	2794	2523	5044
19	4511	4218	3943	3649	3374	3081	2806	2513	2185	4440
	4595	4117	4011	3682	3298	3077	2809	2468	2158	4433
20	8952	8370	7824	7242	6696	6115	5568	4987	4440	8673
	8794	8341	7766	7135	6612	6116	5510	4999	4355	8593
Chi	14.93	15.28	25.71	18.36	23.80	22.37	10.41	20.90	20.81	26.71
Total chi-squared statistic = 378.4 on 360df **Do not reject randomness with p-value = 0.25.**										

Calculations using cards from PartyGaming system

Notes:

a) The above tables show a few cases in which the observed pairs do not match the expected numbers. They are shown in italics (e.g., Ace, Ace for both Player and Dealer). 1 in 20 values can lie outside the 95% confidence limits.

b) '-- ' means Not Applicable.

Comments:

The chi-squared statistic was 378.4 on 360 degrees of freedom (df). The p-value is 0.25 This indicates that the RNG is working correctly.

5. Third and Fourth cards for Player and Dealer

In this section, we test whether the next card drawn has the expected odds. For example, in 8-pack Blackjack, if the player already has a 3 and a 10 (we shall count each of 10,J,Q,K as 10) and the Dealer shows an A, then the probability that the next card is a 10 is 127/413. This is because there are 8 x 4 x 4 = 128 cards worth 10 in the 8 x 52 = 416 cards, while 3 cards have been drawn already and 1 of them is a 10.

Assumptions: For calculations for 3rd and 4th cards, the first 2 cards dealt to Player and Dealer were taken in to account. This procedure does not make the chi-squared test strictly valid, although with 8 packs and 416 cards the differences is very small for this calculation.

5.1 Comparison of Player's third card against expected values

If a Player chooses to draw a third card, the probabilities for the type of the third card will depend on the 3 cards (2 for the player and 1 for the dealer) already drawn. In the table below, shaded values are the observed values and un-shaded values are the expected values in 1 million hands.

Sum of first 2 cards	Player's 3rd card									
	1	2	3	4	5	6	7	8	9	10
1	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
2	52	50	48	47	50	60	44	42	39	193
	45	48	48	48	48	48	48	48	48	193
3	816	863	834	863	917	884	867	891	861	3465
	843	843	870	870	870	870	870	870	870	3481
4	1169	1162	1169	1199	1151	1177	1153	1122	1220	4698
	1150	1156	1149	1176	1176	1176	1176	1175	1175	4705
5	1750	1642	1727	1717	1703	1651	1750	1656	1660	6783
	1678	1676	1676	1677	1703	1703	1703	1703	1703	6815
6	1940	2049	2012	1883	1917	2051	1998	1911	2032	8099
	1976	1974	1982	1974	1975	2001	2001	2001	2001	8006
7	2480	2337	2485	2485	2472	2410	2413	2406	2445	9865
	2437	2430	2429	2429	2430	2435	2457	2457	2456	9831
8	2218	2346	2278	2314	2274	2246	2241	2279	2297	9300
	2297	2276	2275	2279	2275	2275	2296	2302	2302	9211
9	2696	2722	2618	2676	2622	2590	2657	2781	2646	10704
	2683	2654	2654	2654	2654	2654	2654	2681	2682	10732
10	2987	2971	3055	2964	3058	2991	2976	2932	3039	12280
	3034	3005	3005	3005	3007	3005	3005	3005	3031	12136
11	3487	3450	3406	3346	3436	3521	3466	3517	3322	13968
	3472	3443	3443	3442	3442	3442	3442	3443	3442	13886
12	4995	4781	4907	4804	4967	4855	4892	4759	4806	19458
	4887	4804	4867	4869	4871	4877	4861	4861	4861	19451
13	3971	3981	3970	3963	4087	3954	3943	3925	3868	15773
	3973	3976	3906	3961	3962	3962	3951	3951	3951	15816
14	3142	3065	3128	2962	3048	3006	3093	3085	2967	12252
	3070	3075	3076	3015	3062	3062	3057	3051	3051	12215
15	2190	2122	2107	2141	2090	2140	2034	2141	2082	8265
	2110	2115	2115	2115	2071	2104	2097	2096	2096	8392
16	1269	1225	1235	1308	1261	1244	1233	1267	1230	5029
	1259	1262	1262	1262	1262	1232	1251	1254	1249	5003
17	41	51	50	57	58	44	52	49	43	189
	48	49	49	49	49	49	47	48	48	194
18	21	7	10	10	13	11	13	8	11	45
	11	11	11	11	11	11	11	11	11	45
19	7	5	5	10	9	8	10	6	5	32
	7	7	7	7	7	7	7	7	7	29
20	13	12	14	20	11	13	15	18	19	57
	14	14	14	14	14	14	14	14	14	58
Chi-sq 27.72		15.63	10.4	19.97	16.15	12.71	9.69	23.29	17.76	7.3
Total chi-squared statistic = 160.6 on 162df **Do not reject randomness with p-value = 0.52.**										

Calculations using cards from PartyGaming system

Notes:

a) To assess whether the type of the third card is random the following method will be used. Every time a third card is drawn the probability for each type will be calculated. These probabilities will be added up and compared with the actual number of times a third card of each type was drawn.

Example:
A,3,3 already drawn

Probabilities of next card

A	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
0.0750	*0.0775*	*0.0726*	*0.0775*	*0.0775*	*0.0775*	*0.0775*	*0.0775*	*0.0775*	*0.3099*

4,5,10 already drawn

Probabilities of next card

A	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
0.0775	*0.0775*	*0.0775*	*0.0751*	*0.0751*	*0.0775*	*0.0775*	*0.0775*	*0.0775*	*0.3075*

Comments:

The chi-squared statistic was 160.6 on 162 degrees of freedom (df). The p-value is 0.52 This indicates that the RNG is working correctly.

5.2 Comparison of Dealer's third card against expected values

In the table below, shaded values are the observed values and un-shaded values are the expected values in 1 million hands.

Sum of first 2 cards	**Dealer's 3rd card**									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
2	284	311	307	312	350	305	334	350	289	1291
	299	319	319	319	319	319	319	319	319	1277
3	732	739	702	778	748	745	764	798	685	2995
	725	725	748	748	748	748	748	748	748	2995
4	1087	1157	1113	1158	1116	1123	1165	1177	1201	4637
	1132	1129	1131	1154	1154	1154	1153	1154	1153	4617
5	1565	1482	1545	1624	1562	1541	1545	1594	1556	6126
	1535	1531	1531	1533	1556	1556	1556	1555	1556	6226
6	1967	2008	2005	1948	1936	1989	2042	1960	1973	7899
	1967	1962	1963	1962	1964	1987	1987	1987	1987	7953
7	2378	2369	2402	2334	2427	2368	2395	2316	2446	9616
	2379	2373	2373	2373	2372	2376	2398	2398	2398	9600
8	2067	2045	2087	2110	2014	2108	2101	2051	2023	8314
	2083	2054	2054	2054	2053	2053	2079	2079	2079	8322
9	2460	2475	2418	2438	2420	2396	2428	2411	2414	9741
	2445	2418	2415	2415	2415	2414	2417	2441	2441	9770
10	2786	2675	2730	2734	2803	2705	2773	2758	2785	11069
	2771	2744	2743	2740	2742	2741	2742	2743	2766	11073
11	3063	3072	3011	3169	3112	3141	3113	3106	3119	12281
	3109	3082	3082	3081	3078	3078	3080	3080	3080	12425

12	5717	5456	5473	5557	5623	5607	5589	5795	5680	22364
	5635	5537	5606	5606	5605	5604	5603	5603	5603	22442
13	5385	5353	5207	5268	5184	5178	5241	5234	5208	20878
	5270	5265	5171	5241	5241	5240	5238	5238	5238	20978
14	4893	4877	4944	4790	4903	4977	4859	4814	4904	19240
	4889	4883	4883	4789	4859	4859	4860	4856	4857	19450
15	4559	4444	4420	4521	4341	4437	4387	4420	4533	17570
	4458	4453	4453	4453	4359	4430	4429	4429	4427	17726
16	3985	4021	3997	4177	4049	3956	4022	3966	4011	16116
	4045	4041	4041	4042	4041	3946	4018	4018	4017	16076
17	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
18	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
19	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
20	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
Chi-2	11.06	9.75	12.07	17.90	9.61	8.64	4.20	19.65	17.85	8.49

Total chi-squared statistic = 119.2 on 135df

Do not reject randomness with p-value = 0.83.

(Note: Ace was counted as 1 for the purposes of this calculation so there is no soft 17)

Calculations using cards from PartyGaming system

Comments:

The chi-squared statistic was 119.2 on 135 degrees of freedom (df). The p-value is 0.83 This indicates that the RNG is working correctly.

5.3 Comparison of Player's fourth card against expected values

If a Player chooses to draw a fourth card, the probabilities for the type of the fourth card will depend on the 4 cards (3 for the player and 1 for the dealer) already drawn. In the table below, shaded values are the observed values and un-shaded values are the expected values in 1 million hands.

Sum of first 3 cards	**Player's 4th card**									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
2	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
3	4	3	7	2	3	4	3	4	2	19
	3	3	3	3	3	3	3	3	3	15
4	53	45	57	62	60	52	50	67	53	246
	54	55	57	57	57	57	57	57	57	230
5	130	141	132	140	154	143	135	133	146	537
	132	133	137	138	138	138	138	138	138	554
6	249	241	262	244	237	238	250	260	248	1003
	241	243	245	248	250	250	250	250	250	1001

7	317	270	302	284	329	326	316	292	301	1243
	300	299	302	304	307	308	308	308	308	1232
8	224	250	223	229	246	247	249	246	255	993
	242	235	238	241	244	244	245	245	244	979
	302	299	294	292	297	326	302	323	334	1235
	310	298	302	304	307	310	310	310	310	1240
10	464	426	417	400	441	423	433	460	511	1893
	454	440	443	446	448	452	454	454	454	1818
11	607	649	590	574	563	628	584	622	607	2430
	608	592	595	597	600	603	606	608	608	2433
12	992	976	1061	974	1007	1049	988	1002	981	3956
	994	990	991	993	994	997	999	1001	1003	4021
13	1388	1319	1364	1354	1410	1237	1366	1324	1334	5428
	1335	1335	1344	1345	1347	1349	1348	1350	1351	5413
14	1431	1414	1428	1436	1469	1457	1368	1497	1373	5778
	1427	1422	1426	1434	1435	1437	1435	1436	1437	5756
15	1252	1291	1327	1370	1236	1329	1387	1305	1312	5440
	1323	1320	1318	1323	1329	1330	1327	1328	1328	5319
16	1028	1000	1017	1004	959	986	995	970	1018	4043
	1001	999	998	997	1000	1004	1002	1002	1002	4012
17	50	41	36	43	41	40	42	51	31	146
	40	40	40	40	40	39	40	40	40	160
18	8	5	9	3	5	4	3	7	7	26
	5	5	5	5	5	5	5	5	5	23
19	3	4	2	3	2	4	0	2	1	11
	2	2	2	2	2	2	2	2	2	9
20	4	3	0	0	2	2	4	3	4	9
	2	2	2	2	2	2	2	2	2	9
Chi-sq 16.84		15.62	20.69	14.55	19.13	21.18	14.33	13.29	20.09	12.38
Total chi-squared statistic = 168.1 on 153df **Do not reject randomness with p-value = 0.19.**										

Calculations using cards from PartyGaming system

Comments:

The chi-squared statistic was 168.1 on 153 degrees of freedom (df). The p-value is 0.19 This indicates that the RNG is working correctly.

5.4 Comparison of Dealer's fourth card against expected values

In the table below, shaded values are the observed values and un-shaded values are the expected values in 1 million hands.

Sum of first 3 cards	**Dealer's 4th card**									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
2	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--

3	19	27	18	18	20	19	24	15	22	102
	19	22	22	21	22	22	21	22	22	87
4	65	68	71	89	64	83	99	90	70	344
	75	78	80	80	80	80	80	80	80	323
5	188	159	142	175	148	155	181	158	147	680
	157	159	162	165	165	165	165	165	165	661
6	266	287	287	301	290	304	302	273	303	1123
	279	280	284	286	289	289	289	289	289	1157
7	445	434	398	438	453	422	456	447	453	1744
	427	429	432	435	438	440	440	440	440	1763
8	164	183	172	184	170	190	186	179	197	747
	184	175	177	180	183	183	183	183	183	735
9	340	266	318	304	350	317	370	322	352	1238
	324	311	314	317	320	323	323	323	323	1294
10	480	478	491	495	467	461	523	441	457	1909
	481	465	468	471	474	477	480	480	480	1922
11	701	661	641	652	664	658	633	694	662	2646
	668	649	652	655	658	660	664	667	667	2669
12	1452	1545	1431	1424	1509	1515	1464	1468	1533	6039
	1480	1478	1480	1482	1484	1486	1491	1494	1498	6004
13	1998	2001	2066	1936	2024	2048	1980	1963	2051	8104
	2001	1990	2003	2006	2009	2010	2015	2017	2021	8098
14	2442	2430	2455	2468	2408	2424	2402	2428	2466	9739
	2430	2412	2416	2430	2431	2434	2437	2441	2443	9788
15	2773	2770	2821	2775	2827	2823	2738	2740	2845	11491
	2817	2797	2794	2798	2811	2814	2819	2820	2823	11308
16	3153	3156	3097	3077	3233	3201	3112	3099	3184	12529
	3148	3128	3126	3122	3127	3140	3145	3148	3148	12609
17	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
18	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
19	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
20	--	--	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--	--	--
Chi-2	14.07	14.77	13.99	10.90	14.69	6.16	22.00	15.45	11.26	12.15

Total chi-squared statistic = 135.4 on 126df

Do not reject randomness with p-value = 0.27.

(Note: Ace was counted as 1 for the purposes of this calculation so there is no soft 17)

Calculations using cards from PartyGaming system

Comments:

The chi-squared statistic was 135.4 on 126 degrees of freedom (df). The p-value is 0.27 This indicates that the RNG is working correctly.

6. Conclusion

Probabilities for specific events from first two cards for Player and Dealer: These calculations indicated statistical randomness. All observed values were within 95% confidence limits of the theoretical probabilities.

Probabilities for the Dealer's final hand: These calculations indicated statistical randomness. The chi-squared statistic and the p-value were within acceptable limits.

Comparison of Player's and Dealer's first two cards: These calculations indicated statistical randomness. All observed probabilities except one were within 95% confidence limits. 1 in20 values can lie outside the range.

Comparison of Player's and Dealer's first two cards against expected values: These calculations indicated statistical randomness. The chi-squared statistic and the p-value were within acceptable limits.

Comparison of Player's and Dealer's third and fourth cards against expected values: These calculations indicated statistical randomness. The chi-squared statistic and the p-value were within acceptable limits.

In all the above chi-squared tests, none was found to be statistically significant. There is thus no evidence of non-randomness in the sequence of cards drawn. iTech Labs concludes that the Random Number Generator used for the Blackjack game in the PartyGaming system is operating correctly and the cards are being dealt in a fair manner.

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